ALGOR AND THE MACNEAL PROPOSED STANDARD SET OF PROBLEMS TO TEST FINITE ELEMENT ACCURACY

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In this paper we present the results of the tests proposed by MacNeal in the paper[1] "A Proposed Standard set of Problems to Test Finite Element Accuracy" using Algor Release 12 for Windows 98/NT to solve this tests. The Algor Release 12 is available to download free and limited time trial at www.algor.com.

The tests

Patch test for plate

a=0.12; b=0.24; t=0.001; E=1.0x10^6; ν=0.25
Location of inner nodes:

	x	y
1	0.04	0.02
2	0.18	0.03
3	0.16	0.08
4	0.08	0.08

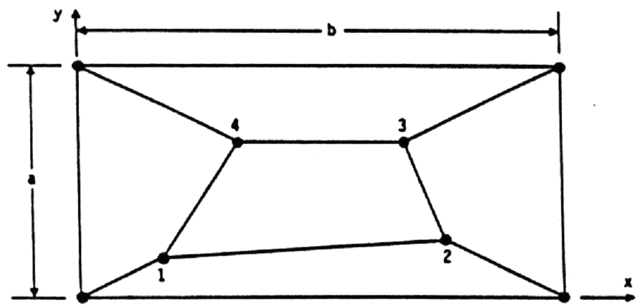


Fig. 1. Patch test for plates.

Membrane plate patch test:

Boundary conditions:	*Theoretical solution:*
$u = 10^{-3}(x+y/2)$	$\gamma_x=\gamma_y=(=10^{-3}$
$v = 10^{-3}(y+x/2)$	$\Phi_x=\Phi_y=1333$; $\vartheta_{xy}=400$

Bending plate patch test:

Boundary conditions:	*Theoretical solution:*
$T = 10^{-3}(x^2+ xy + y^2)$	*Bending moments unit length:* $m_x=m_y=1.111x10^{-7}$
$2_x= 10^{-3}(y+x/2)$	*Surface stresses:* $\Phi_x=\Phi_y=+-0.667$; $\vartheta_{xy}=+-0.200$
$2_y = 10^{-3}(-x-y/2)$	

[1] MACNEAL, R.H., HARDER, R. L.; "A Proposed Standard set of Problems to Test Finite Element Accuracy", *Finite Elements in Analysis and Design 1 (1985) 3-20, North-Holand.*

Patch test for solid

Outer dimensions: unit cube: $E=1.0x10^6$; $\nu=0.25$.
Location of inner nodes:

	x	y	
1	0.249	0.342	0.192
2	0.826	0.288	0.288
3	0.850	0.649	0.263
4	0.273	0.750	0.230
5	0.320	0.186	0.643
6	0.677	0.305	0.683
7	0.788	0.693	0.644
8	0.165	0.745	0.702

Boundary conditions:	*Theoretical solution:*
$u = 10^{-3}(2x+y+z)/2$	$\gamma_x=\gamma_y=\gamma_z=(_{xy}=(_{yz}=(_{zx} =10^{-3}$
$v = 10^{-3}(x+2y+2z)/2$	$\Phi_x=\Phi_y=\Phi_z=2000$; $\vartheta_{xy}=\vartheta_{yz}=\vartheta_{zx}=400$
$w = 10^{-3}(x+y+2z)/2$	

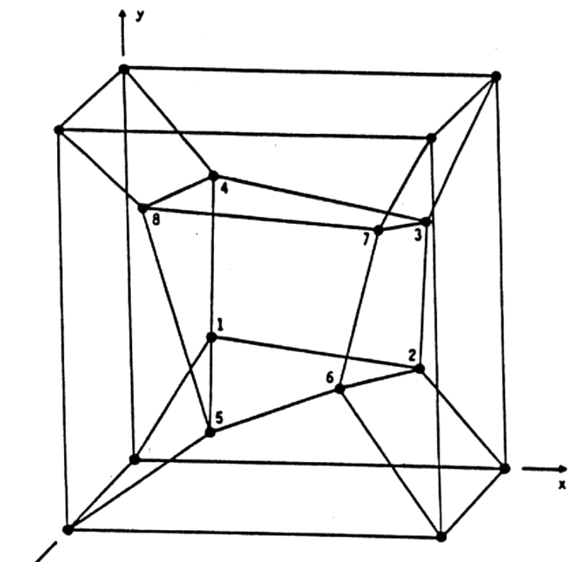


Fig. 2. Patch tests for solids.

Torsion on straight cantilever beam

Length = 6.0; width = 0.2; depth = 0.1; $E = 1.0x10^7$; mesh = 6x1; Loading: unit forces at free end. a) Regular shape elements; b) Trapezoidal shape elements; c) Paralelogram shape elements.
Note: all elements have equal volume.

Theoretical solutions for straight beam problem

Tip load direction	Displacement in direction of load
Extension	3.0×10^{-5}
In-plane shear	0.1081
Out-of-plane shear	0.4321
Twist	0.03208*

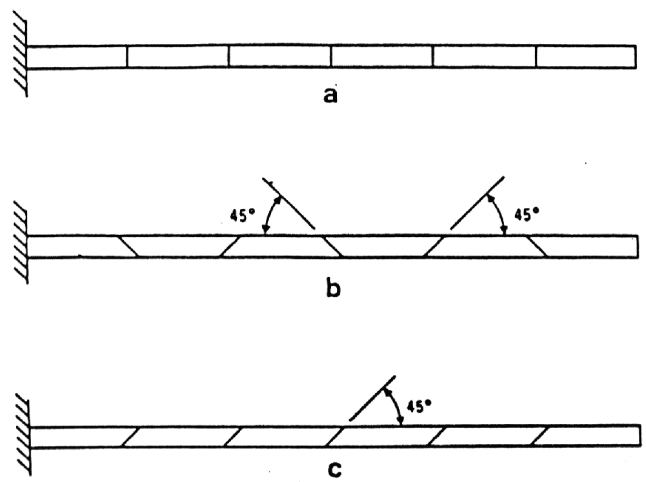


Fig. 3. Straight cantilever beam

* In our opinion the displacement for the problem of torsion of a straight cantilever beam is 0.0034074.
We calculated this value using the expression used by Beer Jonhston[2].
Where:
E = 1.0e7; v = 0.3; G = 3.846144e6;
a = 0.2 ; b = 0.1; L = 6.0
With a/b = 2 results in c1=0.246 e c2=0.229 (table 3.1 pg.282).
The torsion angle ϕ is:

$$\phi = \frac{T * L}{c_2 * a * b^3 * G} = \frac{1.0 * 6.0}{0.229 \ * 0.2 * 0.1^3 * 3.8461 \ .10^{6}} = 0.034061 \quad rad$$

Dx = a/2 * tan ϕ = 0.2/2 * tan 0.034061 = 0.0034074
We generate one FEA model with 20,000 nodes and the dx achieved was 0.003291.

2 BEER, F.P., JOHNSTON, e.r., "Resistência dos Materiais"; McGraw-Hill, 1989,1982, São Paulo, SP.

Curved Beam

Inner radius = 4.12; outer radius = 4.32; arc = 90°; thickness = 0.1; E = $1.0x10^7$; v = 0.25; mesh = 6x1. Loading = unit force at tip.

Theoretical solutions for curved beam problem

Tip load direction	Displacement in direction of load
In-plane shear	0.08734
Out-of-plane shear	0.5022

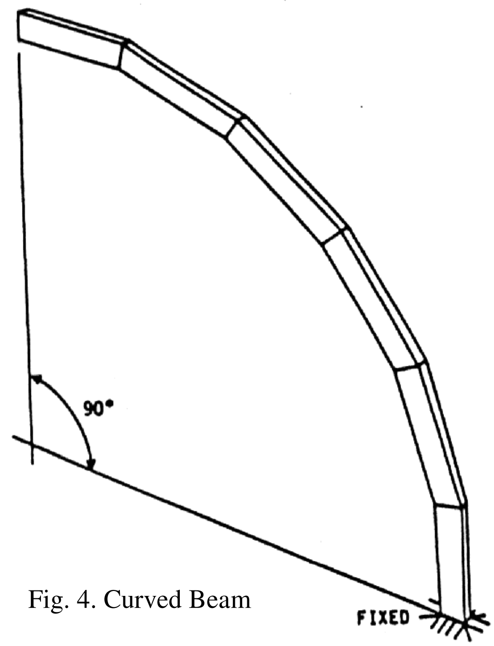


Fig. 4. Curved Beam

Twisted Beam

Length - 120; width -1.1; depth - 0.32; twist - 90^0 (root to tip) E - 29.0 x 10^6; v - 0.22; mesh - 12 x 2. Loading: unit forces at tip.

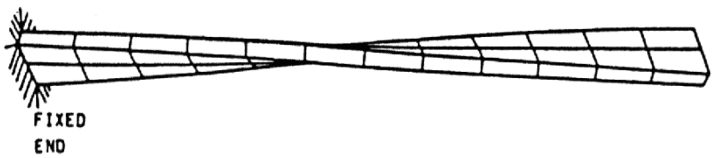


Fig. 5. Twisted Beam

Theoretical solutions for twisted beam problem

Tip load direction	Displacement in direction of load
In-plane shear	0.005424
Out-of-plane shear	0.001754

Rectangular plate

a=2.0; b=2.0 or 10.0; Thickness=0.01; E=$1.7472x10^7$; v = 0.3; boundaries=simply suported or clamped; mesh= NxN(on 1/4 of plate). Loading=uniform pressure. q=10^{-4}, or central load P= $4.0x10^{-4}$.

We used the thickness equal to 0.01 for both plates and bricks, because when using thickness equal to 0.0001 for plates the displacements are large compared to the plate thickness.

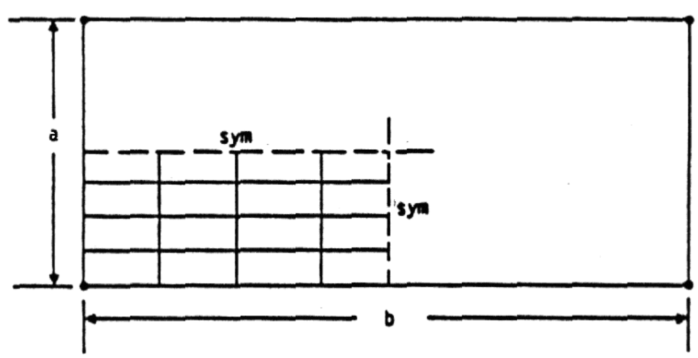


Fig. 5. Rectangular plate

Theoretical solutions for rectangular plate

Boundary supports	Aspect ratio b/a	Displacement at center of plate (10^{-6})	
		uniform pressure	Concentrated force
Simple	1.0	4.062	11.60
Simple	5.0	12.97	16.96
Clamped	1.0	1.26	5.6
Clamped	5.0	2.56	7.23

Scordelis-Lo roof.

Radius - 25.0; length -50.0; thickness -0.25; E - 4.32 x 10^8; v - 0.0; loading - 90.0 per unit area in - Z direction; $u_x = u_z = 0$ on curved edges; mesh: N x N on shaded area.

Theoretical solution

The value for the midside vertical displacement quoted in [5] is 0.3086. Many finite elements converge to a slightly smaller value. We have used the value 0.3024 for normalization of our results.

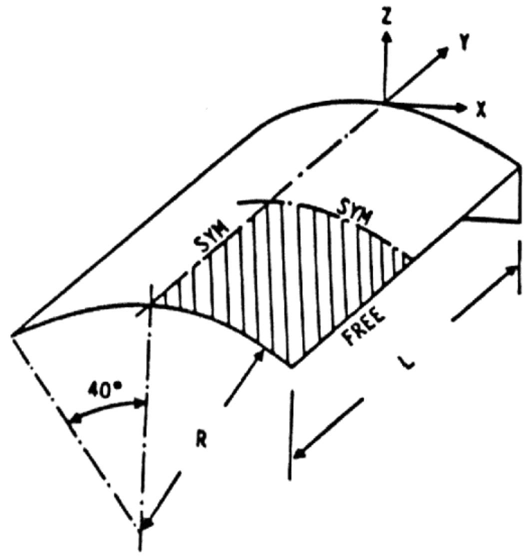


Fig. 6. Scordelis - Lo roof

Thick-walled cylinder.

Thick-walled cylinder . Inner radius = 3.0; outer radius = 9.0; thickness = 1.0; E = 1000; v = 0.49, 0.499, 0.4999; plane strain condition; mesh : 5 x 1 (as shown). Loading: unit pressure at inner radius.

Theoretical solution

Formula for radial displacement:

$$u = \frac{(1+v)pR_1^2}{E(R_2^2 - R_1^2)}\left[R_2^2 / r + (1-2v)r\right]$$

where p = pressure; R_1 = inner radius; R_2 = outer radius

Poisson's ratio	Radial displacement at r = R_1
0.49	5.0399 x 10^{-3}
0.499	5.0602 x 10^{-3}
0.4999	5.0623 x 10^{-3}

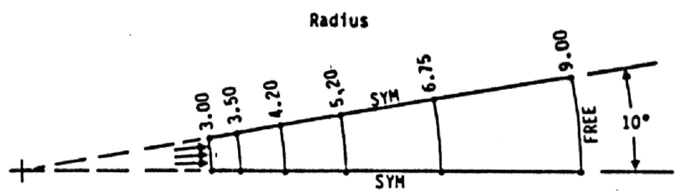
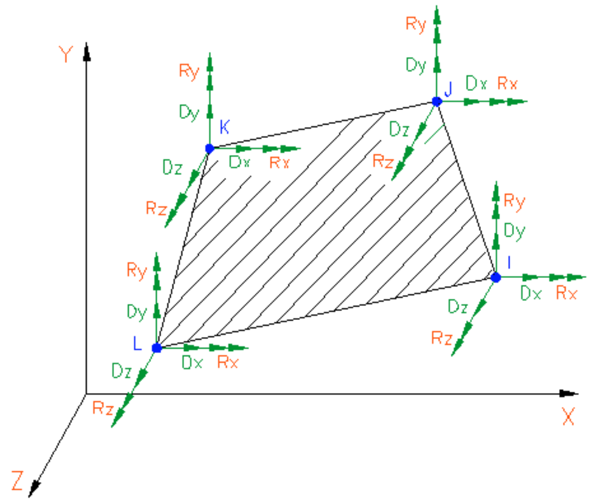


Fig. 7. Thick-walled cylinder

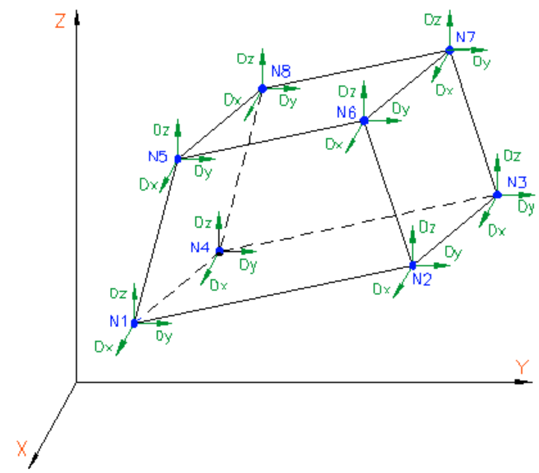
Algor Elements used in this tests

Type 6



Plate/Shell elements are Type 6 elements. These three- or four-node elements are formulated in three-dimensional space. Five degrees-of-freedom are defined for these elements: three translations and two rotations which produce out-of-plane bending. The rotation normal to the plane of the plate is not defined.
Element Formulation Method:
0: QM5 plane stress element and Veubeke plate element boundary element formulation
1: Constrained Linear Strain Triangle (CLST) with Reduced Shear Integration. HCT (Hsieh, Clough and Tocher) plate bending element is used.
2: Same as above but without reduced shear integration.
3: Constant Strain Triangle (CST) with HCT plate bending element.
In this tests are used only method 0 (Veubeke)

Type 5



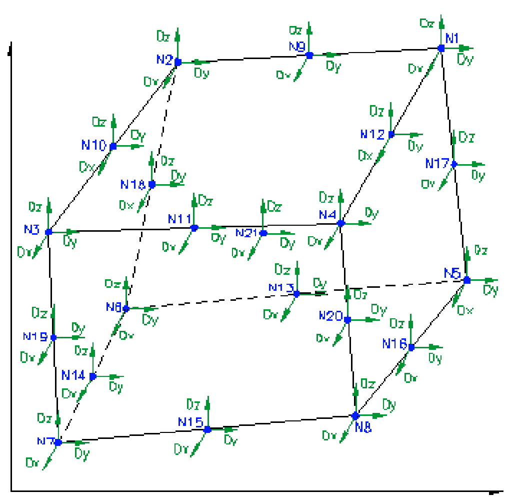
Three-dimensional, solid elasticity elements are Type 5 elements.
These four to eight-node elements are formulated in three-dimensional space and have only three degrees-of-freedom defined per node: the X translation, the Y translation, and the Z translation (see Figures 1 through 6). Isotropic material properties are assumed, and incompatible displacement modes are assumed in the formulation of the element stiffnesses. Pressure, thermal, and uniform inertia loads in three directions are the allowable element based loadings.
In this tests are used 2nd integration order and incompatible mode.

Type 26

Three-dimensional shell elements are Type 26 elements and are 4- to 8-node isoparametric quadrilaterals or 3- to 6-node triangular elements in any 3-D orientation.
In this tests are used only the high-order option with 8 nodes.

Type 25

Three-dimensional solid elements are Type 25 elements. A general 3-D isoparametric element with a variable number of nodes from 8 to 21 can be used. The first 8 nodes are the corner nodes of the element; nodes 9 to 20 correspond to mid-side-nodes; and node 21 is a center node.
In this tests are used only the high-order option with 20 nodes.



Algor test results

Table 1. – Patch test results

	Maximum error in stress			
	Type 6	Type 26	Type 5	Type 25
Constant-stress loading	0.00%	21.65%	0.00%	-
Constant-curvature loading	3.60%	-	N/A	N/A

Table 2 - Results for straight cantilever beam

	Normalized tip displacement in direction of load			
Tip loading direction	Type 6	Type 26	Type 5	Type 25
	(a) *Rectangular elements*			
Extension	0.996	1.005	0.988	1.000
In-plane shear	0.993	0.987	0.978	0.970
Out-of-plane shear	0.984	0.992	0.973	0.961
Twist*	0.567	0.880	0.840	0.851
	(b) *Trapezoidal elements*			
Extension	1.010	1.004	1.005	1.000
In-plane shear	0.052	0.900	0.040	0.886
Out-of-plane shear	0.985	0.947	0.025	0.923
Twist*	0.488	0.927	0.570	0.920
	(c) *Parallelogram elements*			
Extension	1.011	1.004	1.006	1.001
In-plane shear	0.633	0.980	0.615	0.968
Out-of-plane shear	0.985	0.968	0.523	0.942
Twist*	0.705	0.853	1.188	0.788

Table 3. – Results for curved beam

	Normalized tip displacement in direction of load			
Tip loading direction	Type 6	Type 26	Type 5	Type 25
In-plane (vertical)	0.889	1.003	0.738	0.997
Out-of-plane	0.666	0.956	0.700	0.937

Table 4. – Results for twisted beam

	Normalized tip displacement in direction of load			
Tip loading direction	Type 6	Type 26	Type 5	Type 25
In-plane	0.657	0.849	0.980	0.996
Out-of-plane	0.835	7.862	0.977	1.001

Table 5 – Results for rectangular plate simple supports: uniform load

(a) Aspect ratio = 1.0 Normalized lateral deflection at center

Number of nodes spaces per edge of model	Type 6	Type 26	Type 5	Type 25
2	0.870	0.699	0.040	
4	0.965	0.969	0.413	0.991
6	0.984		0.788	
8	0.991	0.994	0.919	0.999

(b) Aspect ratio = 5.0 Normalized lateral deflection at center

Number of nodes spaces per edge of model	Type 6	Type 26	Type 5	Type 25
2	1.087		0.024	
4	1.023	1.002	0.303	1.025
6	1.009		0.722	
8	1.004	0.995	0.917	0.997

Table 6 – Results of rectangular plate clamped supports: concentrated load

(a) Aspect ratio = 1.0 Normalized lateral deflection at center

Number of nodes spaces per edge of model	Type 6	Type 26	Type 5	Type 25
2	0.900			
4	0.966	0.857	0.306	0.822
6	0.984			
8	0.992	0.976	0.824	0.960

(b) Aspect ratio = 5.0 Normalized lateral deflection at center

Number of nodes spaces per edge of model	Type 6	Type 26	Type 5	Type 25
2	0.613		0.006	
4	0.806	0.401	0.083	0.374
6	0.858		0.247	
8	0.883	0.806	0.415	0.782

Table 7 – Results for Scordelis-Lo roof

Normalized vertical deflection at midpoint of free edge

Number of nodes spaces per edge of model	Type 6	Type 26	Type 5	Type 25
2	1.238		0.128	
4	1.005	1.003	0.492	1.004
6	0.985		0.827	
8	0.980	0.996	0.943	1.006
10	0.978			

Table 8 – Results for thick-walled cylinder

	Normalized radial displacement at inner boundary			
Poisson's ratio	Type 6	Type 26	Type 5	Type 25
0.49	1.029	1.097	1.030	1.038
0.499	1.030	1.098	1.034	1.039
0.4999	1.030	1.098	1.098	1.034

Table 9 – Summary of test results for shell elements

Test	Element loading		Element shape	Type 6	Type 26
	In-plane	Out-of-plane			
(1) Patch test	X		Irregular	A	D
(2) Patch test		X	Irregular	B	-
(3) Straight beam, extension	X		All	A	A
(4) Straight beam, bending	X		Regular	A	A
(5) Straight beam, bending	X		Irregular	F	B
(6) Straight beam, bending		X	Regular	A	A
(7) Straight beam, bending		X	Irregular	A	B
(8) Straight beam, twist			All	F	C
(9) Curved beam	X		Regular	C	A
(10) Curved beam		X	Regular	D	B
(11) Twisted beam	X	X	Regular	B	F
(12) Rectangular plate (N = 4)		X	Regular	B	C
(13) Scordelis-Lo roof (N = 4)	X	X	Regular	A	A
(14) Thick-walled cylinder (v = 0.4999)	X		Regular	B	B
Number of failed tests (D's and F's)				3	2

Table 10 – Summary of test results for solid elements

Test	Element shape	Type 5	Type 25
(1,2) Patch test	Irregular	A	-
(3) Straight beam, extension	All	A	A
(4,6) Straight beam, bending	Regular	B	B
(5) Straight beam, bending	Irregular[a]	F	C
(7) Straight beam, bending	Irregular[b]	F	B
(8) Straight beam, twist	All	D	D
(9) Curved beam in-plane loading	Regular	D	A
(10) Curved beam out-of-plane loading	Regular	D	B
(11) Twisted beam	Regular	A	A
(12) Rectangular plate (N = 4)	Regular	F	C
(13) Scordelis-Lo roof (N = 4)	Regular	F	A
(14) Thick-walled cylinder (v = 0.4999)	Regular	B	B
Number of failed tests (D's and F's)		7	1

[a] Bending in plane of irregularity
[b] Bending out of plane of irregularity